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Avoiding Local Minima in the Potential Field Method using Input-to-State Stability

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Abstract

Supported by a novel field definition and recent control theory results, a new method to avoid local minima is proposed. It is formally shown that the system has an attracting equilibrium at the target point, repelling equilibriums in the obstacles centers and saddle points on the borders. Those unstable equilibriums are avoided capitalizing on the established Input-to-State Stability (ISS) property of this multistable system. The proposed modification of the PF method is shown to be effective by simulation for a two variables integrator and then applied to an unicycle-like wheeled mobile robots which is subject to additive input disturbances.

1. Introduction

Path planning is the crucial problem to solve when dealing with navigation for mobile robotics. Very early results on the field [1] date back to the beginning of second half of the last century but in the 80's, when the amount of research increased, results were produced which are used up to the present day [2, 3].

Path planning can be interpreted in two different ways [4]. First, motion planning, in which the path is computed *a-priori* knowing the environment and the robot model to determine a collision-free path. In this case a solution can be evaluated for very complex scenarios but uncertainties (changing) in the models of the environment, or of the robot, could lead to failure. In this category it is possible to find approaches based on Dijkstra or the A* algorithms [5, 6], Potential Field methods [7] and Rapidly exploring Random Trees (RRT) [8, 9]. The second category is represented by the sensor based approaches (*reactive*), in order to avoid the *a-priori* knowledge of the map and deal with unknown conditions; among them it is possible to list the Dynamic Window Approach (DWA) [10], the Velocity Obstacles approach [11], the Virtual Field Histogram (VFH) [12] and its modification VFH+ [13], the last two based on the Potential Field (PF) method [14]. It is straightforward to understand that a combination of the two categories is the best solution to the path planning problem for mobile robots and it is indeed the most adopted [15, 16, 17, 18].

This work is inspired by a recent result [19], in which a notion of Input to State Stability (ISS) for systems evolving in Riemannian manifolds is presented. The method takes into account multiple disconnected invariant sets and it allows the robustness against external disturbances to be evaluated in this complex scenario.

Using this notion, it is hereby presented a reactive obstacle avoidance technique for WMRs based on the PF method.

The main problem with the PF method is the appearance of local minima which block the WMR and prevent to achieve the task ([20] and Section 2 for further details), thus the first contribution of this work is represented by a local minima avoidance technique along with an *ad-hoc* defined field. Starting from the hypothesis of disjoints obstacles, common in literature [16], a twice differentiable PF is designed and the gradient of such a field is used as input for a two variables integrator. As detailed later in the manuscript, under certain assumptions the system is shown to be Input-to-State-Stable (ISS) with respect to decomposable invariants sets [19]. Formally proving the ISS property allows us to escape local minima and to guarantee the global attractiveness of the target point. The singularities are avoided adding a complementary input which plays on the fact that the appearance of any bounded perturbation does not compromise the ISS property [21] and this result is formally proven in the paper.

The designed PF and local minima avoidance technique are applied to drive a unicycle like Wheeled Mobile Robot (WMR) subject to additive input disturbances to a target (*i.e.* the origin). The aim is to have the WMR to track the movement of the 2D particle. The stabilization and tracking problem for non-holonomic WMR has been previously treated in literature [16, 22, 23, 24], often in obstacles free scenario. Here we present 2 approaches: the former one applies an output linearization technique [25, 26] and it is indeed the simplest. The controls obtained for the particle case is applied, with a simple change of coordinates, directly to the WMR control inputs; the inconvenient is that this approach does not allow us to control the robot orientation.

A second controller,and second contribution of this paper, is designed to control both linear velocity and orientation of the WMR. This controller assigns for the linear velocity the norm of the field gradient while the angular velocity command is regulated with a *finite-time* control similar to the one used in [24].

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It is formally shown that the finite time control robustly guarantees the convergence of the robot orientation to the gradient direction and simulations have been carried to illustrate the behaviour.

The contribution of the paper can be summarized in **two** main points:

- A solution to the local minima appearance in the PF method based on [19];
- A *finite time* control able to robustly track the trajectories generated by the designed PF.

Moreover, an experimental part sees a Turtlebot 2 WMR avoiding obstacles in an office-like environment. Usually, obstacle avoidance methods (as the PF one) relied on ultrasonic sensor [27] or infrared ones [28] while the actual trend is to use camera devices or laser range finders; in this work we use a LIDAR device to localize the WMR in a map with unknown obstacles and to realize the avoidance.

The paper is organized as follows: Section 2 presents some related works, Section 3 recalls the results of [19], Section 4 contains the definition of the field, its properties along with the main result. Section 5 shows how to apply the result to an unicycle-like WMR, presents the finite time control, and shows simulations and experiments; the paper ends with a conclusive section 6 in which the authors present also some future directions.

2. Related Works

The Potential Field method was firstly introduced by Khatib in [29] and developed in its generalized modification in [30]. Originally designed for manipulators (other examples can be found in [31] and [32]), it has been modified to drive a mobile robot along a potential field whose minimum is at the target and in which each obstacle generates an additional repellent force which drives the robot away from it. It has been shown that this solution, even though mathematically elegant and quite effective practically, has some drawbacks when special events occur [20]. The main inconvenient with the method is the appearance of *local minima* which block the robot due to particular obstacle configurations. Koditschek *et al.* in [14] proposed a modification of the PF based on *navigation functions*: in a n-dimensional spherical space the adopted field had no other local minima than the target specified, supposing though the complete environment to be known a priori. Other solutions use a harmonic potential field proposed in [33, 34], and the more recent [35], in which the method computes solutions to Laplace's Equation in arbitrary n-dimensional domains to have local minima free field, and results in a weak form of [14]. In [36], a different field formulation and obstacle representation are considered: the potential field includes 2 superquadric functions, one for the obstacle avoidance and one for the approaching which result in an elliptic isopotential contour of the obstacles to model a large variety of shapes. Last flaw of the method is the possibility to miss the target in case of an obstacle too close to it. This problem called *Goals nonreachable with obstacles nearby* (GNRON), treated in [37], deals with the case in which the repulsive force generated by an obstacle close to the target generate a force higher than the attractive one, preventing the robot to accomplish the task. There are also methods which do not eliminate unwanted equilibriums but generate local forces, *Virtual Hill*, to escape the disturbing minimum as in [38].

Within the local planners directly derived from the PF approach, as mentioned above, the VFH method firstly presented in [12] (see also its more recent modifications [13, 39]) represents also a widely used solution to real-time obstacle avoidance. The first experiments ran on WMRs showed the shortcomings inherited after the PF approach: presence of traps and local minima. Thus, hybrid modifications merging global and local planners, like VFH+, were proposed: starting from a grid map, evaluates the PF at each iteration for a subset of active cells of the map, builds an obstacle histogram and reduces it to a polar form to finally compute the velocity commands. Many other modifications [40, 41, 42, 43, 44] have been proposed in order to overcome all the cited shortages. In [42], a new smooth repulsive force for the field is proposed applying the *i-PID* control method but no proof is given about actions taken to avoid local minima. The paper [43] studies just some aspects of the potential field method as local planner focusing on the repulsive field and not addressing the local minima problem. The work [41] is the one more similar to the result of the authors and it is based on a potential field which is not smooth. In [40] the PF method is applied to drive a group of WMR to a goal; the design of the PF is made to accomplish this task avoiding local minima but very simple and standard control techniques are applied (PI control).

3. Preliminaries

For an n-dimensional C^2 connected and orientable Riemannian manifold M without boundary (it is assumed here that M can be embedded in a Euclidean space, thus $0 \in M$), let the map $f : M \times \mathbb{R}^m \to T_xM$ be of class C^1 (T_xM is the tangent space), and consider a nonlinear system of the following form:

$$\dot{x}(t) = f(x(t), d(t)) \tag{1}$$

where the state $x \in M$ and $d(t) \in \mathbb{R}^m$. The input $d(\cdot)$ is a locally essentially bounded and measurable signal for $t \geq 0$. We denote by $X(t, x; d(\cdot))$ the uniquely defined solution of (1) at time t fulfilling $X(0, x; d(\cdot)) = x$. Together with (1) we will analyze its unperturbed version:

$$\dot{x}(t) = f(x(t), 0). \tag{2}$$

A set $S \subset M$ is invariant for the unperturbed system (2) if $X(t, x; 0) \in S$ for all $t \in \mathbb{R}$ and for all $x \in S$. For a compact set $S \subset M$ define the distance to the set $|x|_S = \min_{a \in S} \delta(x, a)$ from a point $x \in M$, where the symbol $\delta(x_1, x_2)$ denotes the Riemannian distance ([45]) between x_1 and x_2 in M, $|x| = |x|_{\{0\}}$ for $x \in M$ or a usual euclidean norm of a vector $x \in \mathbb{R}^n$. For a signal $d : \mathbb{R} \to \mathbb{R}^m$ the essential supremum norm is defined as $\|d\|_\infty = \text{ess sup}_{t \geq 0} |d(t)|$.

3.1. Decomposable sets

Let $\Lambda \subset M$ be a compact invariant set for (2).

Definition 1. [46] A decomposition of Λ is a finite and disjoint family of compact invariant sets $\Lambda_1, \ldots, \Lambda_k$ such that

$$\Lambda = \bigcup_{i=1}^{k} \Lambda_i.$$

For an invariant set Λ, its attracting and repulsing subsets are defined as follows:

$$W^s(\Lambda) = \{x \in M : |X(t,x,0)|_\Lambda \to 0 \text{ as } t \to +\infty\},$$
$$W^u(\Lambda) = \{x \in M : |X(t,x,0)|_\Lambda \to 0 \text{ as } t \to -\infty\}.$$

Define a relation on $\mathcal{W} \subset M$ and $\mathcal{D} \subset M$ by $\mathcal{W} \prec \mathcal{D}$ if $W^s(\mathcal{W}) \cap W^u(\mathcal{D}) \neq \emptyset$.

Definition 2. [46] Let $\Lambda_1, \ldots, \Lambda_k$ be a decomposition of Λ, then

1. An r-cycle ($r \geq 2$) is an ordered r-tuple of distinct indices $i_1, \ldots, i_r$ such that $\Lambda_{i_1} \prec \ldots \prec \Lambda_{i_r} \prec \Lambda_{i_1}$.
2. A 1-cycle is an index i such that $[W^u(\Lambda_i) \cap W^s(\Lambda_i)] - \Lambda_i \neq \emptyset$.
3. A filtration ordering is a numbering of the Λ_i so that $\Lambda_i \prec \Lambda_j \Rightarrow i \leq j$.

As we can conclude from Definition 2, existence of an r-cycle with $r \geq 2$ is equivalent to existence of a heteroclinic cycle for (2) [47]. And existence of a 1-cycle implies existence of a homoclinic cycle for (2) [47].

Definition 3. The set $\mathcal{W}$ is called decomposable if it admits a finite decomposition without cycles, $\mathcal{W} = \bigcup_{i=1}^{k} \mathcal{W}_i$, for some non-empty disjoint compact sets $\mathcal{W}_i$, which form a filtration ordering of $\mathcal{W}$, as detailed in definitions 1 and 2.

Let a compact set $\mathcal{W} \subset M$ be containing all α- and ω-limit sets of (2) [48].

3.2. Robustness notions

The following robustness notions for systems in (1) have been introduced in [19].

Definition 4. We say that the system (1) has the practical asymptotic gain (pAG) property if there exist $\eta \in \mathcal{K}_\infty$ [1]and a non-negative real q such that for all $x \in M$ and all measurable essentially bounded inputs $d(\cdot)$ the solutions are defined for all $t \geq 0$ and the following holds:

$$\limsup_{t \to +\infty} |X(t,x;d)|_{\mathcal{W}} \leq \eta(\|d\|_\infty) + q. \quad (3)$$

If $q = 0$, then we say that the asymptotic gain (AG) property holds.

[1]A continuous function $h : [0,a) \to [0,\infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $h(0) = 0$; it is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $h(r) \to \infty$ as $r \to \infty$ [21].

Definition 5. We say that the system (1) has the limit property (LIM) with respect to $\mathcal{W}$ if there exists $\mu \in \mathcal{K}_\infty$ such that for all $x \in M$ and all measurable essentially bounded inputs $d(\cdot)$ the solutions are defined for all $t \geq 0$ and the following holds:

$$\inf_{t \geq 0} |X(t,x;d)|_{\mathcal{W}} \leq \mu(\|d\|_\infty).$$

Definition 6. We say that the system (1) has the practical global stability (pGS) property with respect to $\mathcal{W}$ if there exist $\beta \in \mathcal{K}_\infty$ and $q \geq 0$ such that for all $x \in M$ and all measurable essentially bounded inputs $d(\cdot)$ the following holds for all $t \geq 0$:

$$|X(t,x;d)|_{\mathcal{W}} \leq q + \beta(\max\{|x|_{\mathcal{W}}, \|d\|_\infty\}).$$

It has been shown in [19] that to characterize (3) in terms of Lyapunov functions the following notion is appropriate:

Definition 7. A C^1 function $V : M \to \mathbb{R}$ is a practical ISS-Lyapunov function for (1) if there exists $\mathcal{K}_\infty$ functions $\alpha_1, \alpha_2, \alpha$ and γ, and scalar $q \geq 0$ and $c \geq 0$ such that

$$\alpha_1(|x|_{\mathcal{W}}) \leq V(x) \leq \alpha_2(|x|_{\mathcal{W}} + c),$$

the function V is constant on each $\mathcal{W}_i$ and the following dissipative property holds:

$$DV(x)f(x,d) \leq -\alpha(|x|_{\mathcal{W}}) + \gamma(|d|) + q.$$

If the latter inequality holds for $q = 0$, then V is said to be an ISS-Lyapunov function.

Notice that the existence of α_2 and c follows (without any additional assumptions) by standard continuity arguments.

The main result of [19] connecting these robust stability properties is stated below:

Theorem 1. *Consider a nonlinear system as in (1) and let a compact invariant set containing all α and ω limit sets of (2) $\mathcal{W}$ be decomposable (in the sense of Definition 3). Then the following facts are equivalent.*

1. *The system admits an ISS Lyapunov function;*
2. *The system enjoys the AG property;*
3. *The system admits a practical ISS Lyapunov function;*
4. *The system enjoys the pAG property;*
5. *The system enjoys the LIM property and the pGS.*

A system in (1), for which this list of equivalent properties is satisfied, is called ISS with respect to the set $\mathcal{W}$ [19].

4. Potential field method with static obstacles

First, let us consider a simplified model of a mobile agent represented by doubled integrator dynamics:

$$\begin{aligned} \dot{x} &= u_x, \\ \dot{y} &= u_y, \end{aligned} \quad (4)$$

where $x \in \mathbb{R}$ and $y \in \mathbb{R}$ are the coordinates of the agent in the plane, $z = [x\ y]^T$, $u_x \in \mathbb{R}$ and $u_y \in \mathbb{R}$ are the corresponding controls. It is necessary to design the controls u_x, u_y providing the agent regulation to the origin under avoidance of collisions with isolated point-wise obstacles, which are defined by their coordinates $\zeta_i = (x_i, y_i)$ and safe distances d_i around them for $i = 1, \ldots, N$, where $N > 0$ is a finite number of obstacles. We will assume that $|\zeta_i - \zeta_j| > \max\{d_i, d_j\}$ and $|\zeta_i| > d_i$ for all $1 \le i \ne j \le N$, *i.e.* the obstacles are separated and the origin is not occupied by an obstacle.

Applying the potential field method, the controls can be designed proportional to the "forces" generated by the total potential U. U is the sum of a repulsion potential U_r with respect to the obstacles and attraction potential U_a with respect to the origin[25, 14]. In this work we will use the results presented in the previous section to design the agent dynamics that is C^1 and ISS with respect to the set $\mathcal{W}$ composed by equilibriums, among them the equilibrium at the origin is attractive, the equilibriums related to the obstacles are repulsing, while ones corresponding to the local extrema are saddle. Next, applying specially designed small perturbations to that ISS system we will avoid the unstable equilibriums.

To design the attraction potential U_a we impose the constraints: it has to be twice continuously differentiable with respect to x and y, and its gradient to be globally bounded. The following potential yields these constraints:

$$U_a(z) = \begin{cases} |z|^2 & \text{if } |z| \le \upsilon, \\ |z| & \text{if } |z| \ge \Upsilon, \\ \lambda(|z|)|z|^2 + [1 - \lambda(|z|)]|z| & \text{otherwise}, \end{cases}$$

$$\lambda(s) = \left(\frac{2s^3 - 3(\upsilon + \Upsilon)s^2 + 6\Upsilon\upsilon s + \Upsilon^2(\Upsilon - 3\upsilon)}{\Upsilon^2(\Upsilon - 3\upsilon) + \upsilon^2(3\Upsilon - \upsilon)} \right)^2,$$

where $0 < \upsilon < \Upsilon < +\infty$ are the design parameters. Thus, the potential U_a is quadratic in z close to the origin, it has a linear growth rate far enough and the function λ ensures a smooth transition between these zones.

The repulsion potential U_r must be also twice continuously differentiable with respect to x and y, and it has to be active only on a small zone around the obstacle (the agent can detect the obstacle presence only locally in an uncertain environment in a robotic application, for example):

$$U_r(z) = \alpha \sum_{i=1}^{N} \max\{0, d_i^2 - |z - \zeta_i|^2\}^2,$$

where $\alpha > 0$ is a tuning parameter.

The total potential U, Fig.1, has the form:

$$U(z) = U_a(z) + U_r(z), \tag{5}$$

with the gradient

$$\nabla_z U(z) = \nabla_z U_a(z) + \nabla_z U_r(z),$$

$$\nabla_z U_a(z) = \begin{cases} 2z & \text{if } |z| \le \upsilon, \\ z|z|^{-1} & \text{if } |z| \ge \Upsilon, \\ \varphi(z) & \text{otherwise}, \end{cases}$$

$$\nabla_z U_r(z) = -4\alpha \sum_{i=1}^{N} (z - \zeta_i) \max\{0, d_i^2 - |z - \zeta_i|^2\},$$

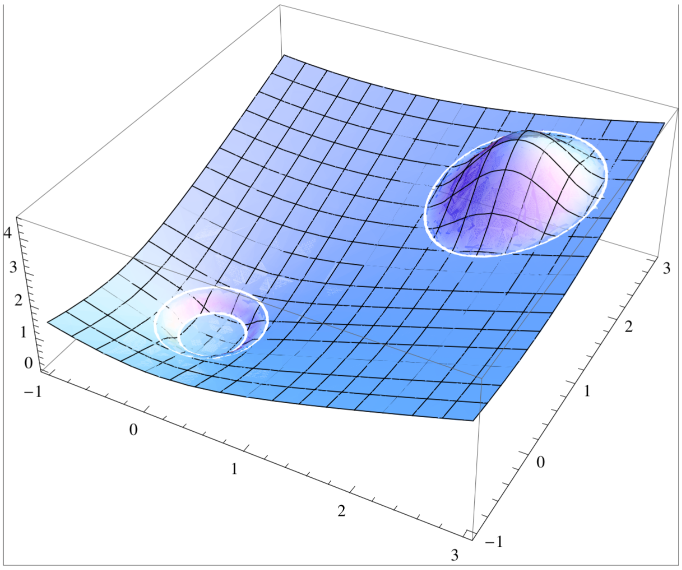


Figure 1: The continuous field in the case of a single obstacle with $v = 0.3$, $\Psi = 0.5$, $\zeta_1 = (2, 2)$, $d_1 = 0.8$ and $\alpha = 4$

where $\varphi(z) = \nabla_z \left(\lambda(|z|)|z|^2 + [1 - \lambda(|z|)]|z| \right)$ is the corresponding C^1 function ensuring a continuous transition between $2z$ and $z|z|^{-1}$. Note that by construction $\nabla_z U(z)$ is a C^1 function of z. As usual in the potential field method we assign:

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = -\nabla_z U(z) + v, \tag{6}$$

where $v \in \mathbb{R}^2$ is an auxiliary bounded input to be designed later, then the closed-loop system (4), (6) takes the gradient form:

$$\dot{z} = -\nabla_z U(z) + v. \tag{7}$$

Next, we are going to show that for $v = 0$ this system has an attracting equilibrium at the origin, repulsing equilibriums in a vicinity of ζ_i for each $i = 1, \ldots, N$ and a saddle equilibrium in the border of the repulsion zone around ζ_i for each $i = 1, \ldots, N$. Therefore, a compact invariant set $\mathcal{W}$ containing all α- and ω-limit sets of (7) for $v = 0$ is decomposable in the sense of Definition 3, and that Theorem 1 can be applied to establish ISS with respect to the set $\mathcal{W}$ for the input v.

4.1. Equilibrium at the origin

Under the restrictions $|\zeta_i| > d_i$ for all $1 \le i \le N$, the system (7) is reduced to

$$\dot{z} = -2z$$

for $|z| \le \tilde{\upsilon}$ for some $0 < \tilde{\upsilon} \le \upsilon$, which is obviously locally attractive. For simplicity of presentation below we will assume that the constants υ and Υ are selected in a way to provide $|\zeta_i| \ge \Upsilon + d_i$ for all $1 \le i \le N$.

4.2. Equilibriums around the obstacles

Since the obstacles are separated from one another and from the origin, around each obstacle the system (7) takes a reduction ($|z| \ge \Upsilon$):

$$\dot{z} = -z|z|^{-1} + 4\alpha(z - \zeta_i) \max\{0, d_i^2 - |z - \zeta_i|^2\} + v$$

for some $1 \le i \le N$. Clearly, if $d_i < |z - \zeta_i|$ then $\dot{z} = -z|z|^{-1}$ and there is no equilibrium, thus we may restrict attention to the case $|z - \zeta_i| \le d_i$ and

$$\dot{z} = -z|z|^{-1} + 4\alpha(z - \zeta_i)(d_i^2 - |z - \zeta_i|^2) + v.$$

On this set the equilibriums of (7) satisfy the vector equation

$$z = 4\alpha(z - \zeta_i)(d_i^2 - |z - \zeta_i|^2)|z|$$

or the corresponding scalar equation

$$|z|^2 - 8\alpha(d_i^2 - |z - \zeta_i|^2)|z|\, z^T(z - \zeta_i)$$
$$+16\alpha^2|z - \zeta_i|^2(d_i^2 - |z - \zeta_i|^2)^2|z|^2 = 0.$$

Introducing parametrization $z = \kappa\zeta_i + \eta$, where $\kappa \in \mathbb{R}$ and $\eta \in \mathbb{R}^2$, and substituting it in the last equation it is tedious but straightforward to obtain that for any $|\eta| \neq 0$ the equality is not satisfied. Therefore setup $\eta = 0$, then under substitution $z = \kappa\zeta_i$ we have

$$\kappa^2|\zeta_i|^2\{1 - 4\alpha[d_i^2 - (\kappa - 1)^2|\zeta_i|^2]|\zeta_i|(\kappa - 1)\}^2 = 0,$$

the equation for equilibriums is reduced to

$$1 - 4\alpha[d_i^2 - s^2|\zeta_i|^2]|\zeta_i|s = 0$$

for $s = \kappa - 1$, or

$$s^2 - \frac{d_i^2}{|\zeta_i|^2}s + \frac{1}{4\alpha|\zeta_i|^3} = 0$$

that is a depressed cubic equation, which by the Cardano's method has only real roots if

$$\alpha d_i^3 > \frac{3\sqrt{3}}{8}. \tag{8}$$

Next, by the Routh–Hurwitz stability criterion the equation has 2 roots with positive real parts. Therefore, for $|z - \zeta_i| \le d_i$ the system (7) has two equilibriums $z_0^{i,1}$ and $z_0^{i,2}$ under the condition (8). The Cardano's method also provides the expressions of exact solutions and, hence, the coordinates of the equilibriums $z_0^{i,1}$, $z_0^{i,2}$ (not given here for compactness, both equilibriums are located farther from the origin than the obstacle ζ_i on the line connecting the origin and the point (x_i, y_i)). Finally, the system (7) is continuously differentiable, then the linearization shows that the equilibrium $z_0^{i,1}$ (closer to ζ_i) is purely repulsing, and another one $z_0^{i,2}$ is saddle (the corresponding local minimum).

To evaluate the zone of repulsion around ζ_i a Lyapunov function for linearization around $z_0^{i,1}$ can be used, or let us consider a Lyapunov function $V(e) = |e|^2$ for $e = z - \zeta_i$ and $v = 0$:

$$\begin{aligned}\dot{V} &= 2e^T[-z|z|^{-1} + 4\alpha e(d_i^2 - |e|^2)]\\ &= -2e^T z|z|^{-1} + 8\alpha V(d_i^2 - V)\\ &\ge -2|e| + 8\alpha V(d_i^2 - V).\end{aligned}$$

Note that $|e| = \sqrt{V}$ then

$$\dot{V} \ge [4\alpha\sqrt{V}(d_i^2 - V) - 1]2\sqrt{V}.$$

The Cardano's method can be used to find the solutions of the equation $4\alpha\sqrt{V}(d_i^2 - V) = 1$, which determines the sign definiteness of $\dot{V}$. The expression in the square brackets $4\alpha\sqrt{V}(d_i^2 - V) - 1$ reaches its maximum $\frac{2}{\sqrt{3}}\left(\frac{8}{3\sqrt{3}}\alpha d_i^3 - 1\right)d_i$ for $V = \frac{1}{3}d_i^2$, which is positive if the condition (8) is fulfilled (note that since the value of d_i is constrained by the physical dimensions of the agent, then (8) is a condition for α to satisfy). Thus the repulsion zone around the obstacle exists and it can be easily estimated.

4.3. Robustness with respect to v

The conditions on existence of the equilibriums, established above, are as follows:

Assumption 1. Let the condition (8) be satisfied, $|\zeta_i - \zeta_j| > \max\{d_i, d_j\}$ and $|\zeta_i| \ge \Upsilon + d_i$ for all $1 \le i \neq j \le N$.

Now we would like to show that the set $\mathcal{W} = \{\{0\}, z_0^{1,1}, z_0^{1,2}, \ldots, z_0^{i,1}, z_0^{i,2}, \ldots, z_0^{N,1}, z_0^{N,2}\}$, which is composed by the equilibrium at the origin and N pairs of equilibriums $z_0^{i,1}, z_0^{i,2}$ associated with each obstacle, contains all α- and ω-limit sets of (7) for $v = 0$ and it is decomposable in the sense of Definition 3. The system (7) has a Lyapunov function $U(z)$, by construction $\alpha_1(|z|) \le U(z) \le \alpha_2(|z|)$ for all $z \in \mathbb{R}^2$ and some $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, whose derivative has the form:

$$\begin{aligned}\dot{U} &= -|\nabla_z U(z)|^2 + \nabla_z^T U(z)v\\ &\le -0.5|\nabla_z U(z)|^2 + 0.5|v|^2\end{aligned} \tag{9}$$

and the total potential stops to decrease for $v = 0$ only on the set where $\nabla_z U(z) = 0$, but by consideration above it is $\mathcal{W}$: *i.e.* there exist $\gamma_1, \gamma_2 \in \mathcal{K}_\infty$ such that $\gamma_1(|z|_{\mathcal{W}}) \le |\nabla_z U(z)| \le \gamma_2(|z|_{\mathcal{W}})$ for all $z \in \mathbb{R}^2$. There is no cycle in the decomposition of $\mathcal{W}$ due to the same property $\dot{U} \le 0$ for $v = 0$ (indeed the obstacles are separated and to pass from one saddle equilibrium around the obstacle ζ_i to another one around ζ_j it is necessary to cross the zone where $\nabla_z U(z) = \nabla_z U_a(z)$ and $\dot{U} < 0$, therefore a trajectory cannot return back). Thus, $\mathcal{W}$ is decomposable and contains all α- and ω-limit sets of (7) for $v = 0$. Further,

$$\dot{U} \le -0.5\gamma_1^2(|z|_{\mathcal{W}}) + 0.5|v|^2,$$

then U is an ISS Lyapunov function and by Theorem 1 the following result has been proven.

Lemma 2. *Under Assumption 1 the system (7) is ISS with respect to the set $\mathcal{W}$ for the input v.*

4.4. Design of the input v to escape local minima

The advantage of the ISS property is that appearance of any bounded disturbance v does not lead to the system instability. In our case the total potential function U is also an ISS Lyapunov function for the system (7). If $v = 0$ and the agent in (7) is approaching an unstable equilibrium, then according to the expression of $\dot{U}$ the velocity of the agent is decreasing proportionally to $|\nabla_z U(z)|$. Thus, if $|\nabla_z U(z)| \le \epsilon$ for some predefined $\epsilon > 0$ and we are far from the origin, it can be a signal of closeness to a saddle equilibrium, then an input $v \neq 0$ can be generated to shift the movement direction.

The input v must be selected bounded and pushing the system in an arbitrary direction with a uniform distribution, by ISS property the solutions asymptotically will stay close to $\mathcal{W}$ and it is possible to show that the origin will be globally attractive. However, using the Lyapunov function U the input v always can be designed in order to additionally guarantee a decreasing of U. From (9)

$$v = \begin{cases} \rho \begin{bmatrix} \nabla_y U(z) \\ -\nabla_x U(z) \end{bmatrix} & \text{if } |\nabla_z U(z)| \leq \epsilon \text{ and } |z| > \upsilon, \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

$$\rho = \operatorname{sgn}\left(y - \frac{y_i}{x_i}x\right), \; i = \arg \inf_{1 \leq j \leq N} |z - \zeta_j|,$$

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{if } s \geq 0, \\ -1 & \text{otherwise} \end{cases}$$

ensures that $\dot{U} \leq 0$ for all $t \geq 0$ ($\dot{U} = -|\nabla_z U(z)|^2$ while $v \neq 0$) and for $U = 0$ as well, and for the case of agent velocity dangerous decreasing ($|\nabla_z U(z)| \leq \epsilon$) far from the origin ($|z| > \upsilon$) the proposed input v generates an orthogonal disturbance to the current direction of movement. The variable ρ defines the orientation of this orthogonal perturbation, in (10) it points out from the line connecting the origin and the point (x_i, y_i) (that is the coordinate of the closest obstacle) and where we have the unstable equilibriums.

Theorem 3. *Under Assumption 1 the system (7) with the avoidance control (10) has the origin attractive from all initial conditions* $z(0) \notin \mathcal{W} \setminus \{0\}$.

Usually, for a robotic application, it is assumed that the robot starts in the collision-free conditions, *i.e.* $z(0) \notin \mathcal{D} = \cup_{i=1}^{N} \mathcal{D}_i$ where $\mathcal{D}_i = \{z \in \mathbb{R}^2 : |z - \zeta_i| \leq d_i\}$. Therefore, in this case definitely $z(0) \notin \mathcal{W} \setminus \{0\}$ since $\mathcal{W} \setminus \{0\} \subset \mathcal{D}$.

Proof. Considering the ISS Lyapunov function U for the system (7) with the avoidance control (10) we obtain:

$$\dot{U} = -|\nabla_z U(z)|^2$$

since $\nabla_z^T U(z) v = 0$ always. In addition, by construction v shifts the system trajectories out from the line $y = \frac{y_i}{x_i}x$ that contains the unwanted equilibriums $z_0^{i,1}, z_0^{i,2}$, then the only point to stop is the origin.

Formally the control (10) does not use ISS property of the set $\mathcal{W}$, it is designed from a pure Lyapunov approach. (10) is modified as follows in order to make the attractiveness of the origin global:

$$v = \begin{cases} \rho \frac{\varepsilon}{|z|} \begin{bmatrix} y \\ -x \end{bmatrix} & \text{if } |\nabla_z U(z)| \leq \epsilon \text{ and } |z| > \upsilon, \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

$$\rho = \operatorname{sgn}\left(y - \frac{y_i}{x_i}x\right), \; i = \arg \inf_{1 \leq j \leq N} |z - \zeta_j|,$$

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{if } s \geq 0, \\ -1 & \text{otherwise} \end{cases}$$

where $\varepsilon > 0$ is a design parameter. It is easy to check that $v^T z = 0$ for all $z \in \mathbb{R}^2$ and $|v| = \varepsilon$ if $|\nabla_z U(z)| \leq \epsilon$ and $|z| > \upsilon$ in (11).

Theorem 4. *Under Assumption 1 the system (7) with the avoidance control (11) has the origin globally attractive provided that $\varepsilon > 2\epsilon$ and $\epsilon > 0$ is selected sufficiently small.*

Proof. From Lemma 2 the system (7) is ISS with respect to the set $\mathcal{W}$ for the input v. By Theorem 1 we known that in this case all solutions in the system remain bounded since $|v| \leq \varepsilon$, and due to AG property we have

$$\limsup_{t \to +\infty} |z(t)|_{\mathcal{W}} \leq \eta(\varepsilon)$$

for some $\eta \in \mathcal{K}$. If the value of ϵ has been selected sufficiently small, then the set $\mathcal{A} = \{z \in \mathbb{R}^2 : |z|_{\mathcal{W}} \leq \eta(\varepsilon)\}$ is a union of separated sets $\mathcal{A}_i^1$ and $\mathcal{A}_i^2$ contained only one equilibrium point $z_0^{i,1}$ and $z_0^{i,2}$ respectively, and a neighborhood $\mathcal{A}_0$ of the origin, *i.e.* $\mathcal{A} = \mathcal{A}_0 \cup \bigcup_{i=1}^{N} (\mathcal{A}_i^1 \cup \mathcal{A}_i^2)$. In $\mathcal{A}_0$ the system is converging to the origin. Assume that $|z(t)|_{\mathcal{W}} \in \mathcal{A}_i^1$ or $|z(t)|_{\mathcal{W}} \in \mathcal{A}_i^2$ for some $1 \leq i \leq N$, then for $|\nabla_z U(z)| \leq \epsilon$ and $|z| > \upsilon$ the input v is always acting from $z_0^{i,1}, z_0^{i,2}$ by construction, then U is strictly decreasing. Indeed, for all cases $\nabla_z U_a(z)$ is proportional to z, then $v^T \nabla_z U_a(z) = 0$ for all $z \in \mathbb{R}^2$. Next, $\nabla_z U_r(z) = 4\alpha(z - \zeta_i)(d_i^2 - |z - \zeta_i|^2)$ for $z \in \mathcal{D}_i$, then $v^T \nabla_z U_r(z) = -4\alpha(d_i^2 - |z - \zeta_i|^2) v^T \zeta_i$ where $4\alpha(d_i^2 - |z - \zeta_i|^2) \geq 0$ for $z \in \mathcal{D}_i$. Due to selection of ρ we have $v^T \zeta_i > 0$, then

$$\dot{U} = -|\nabla_z U(z)|^2 + v^T \nabla_z U(z)$$

with $v^T \nabla_z U(z) \leq 0$ for all $|\nabla_z U(z)| \leq \epsilon$ and $z \in \mathcal{D}_i$. Therefore, U is not increasing. Note that $v^T \nabla_z U(z) = 0$ only if $d_i^2 = |z - \zeta_i|^2$, *i.e.* z belongs to the border of $\mathcal{D}_i$. By selection ϵ (and ε) sufficiently small it is possible to ensure that intersections of $\mathcal{A}_i^1$ and $\mathcal{A}_i^2$ with the set where $|\nabla_z U(z)| \leq \epsilon$ belongs to the interior of $\mathcal{D}_i$, then $v^T \nabla_z U(z) < 0$ always for all $|\nabla_z U(z)| \leq \epsilon$ and $|z| > \upsilon$, thus U is strictly decreasing to zero. In addition, from (7) $|\dot{z}|^2 = (v - \nabla_z U(z))^T (v - \nabla_z U(z)) = |\nabla_z U(z)|^2 - 2v^T \nabla_z U(z) + |v|^2$ and for $|\nabla_z U(z)| \leq \epsilon$ and $|z| > \upsilon$ we have $|\dot{z}|^2 \geq -2\varepsilon\epsilon + \varepsilon^2 > 0$, then there is no new equilibrium point induced by v (as can be seen also in Fig. 2) .

4.5. More complex situations

Of course in reality the assumption about separation between obstacles may be not satisfied, but even for this case the approach can be easily extended. Application of perturbation v

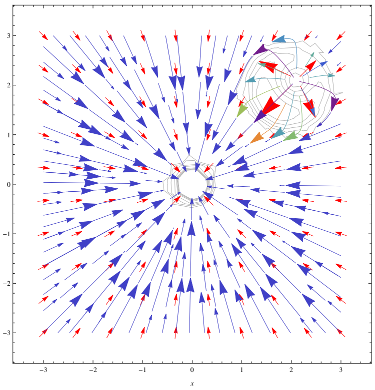


Figure 2: Gradient Lines in the case of a single obstacle with $\nu = 0.3$, $\Psi = 0.5$, $\zeta_1 = (2,2)$, $d_1 = 0.8$,$\alpha = 4$ and $\epsilon = 0.1$

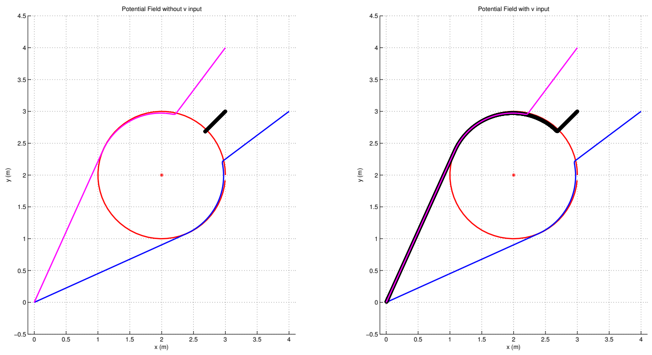


Figure 3: The results of the system (7) simulation

with the amplitude ε do not destroy boundedness of the system trajectories by ISS property. If ε has been selected sufficiently small, then asymptotically $z(t)$ enters $\mathcal{A}$, as it has been defined above, whose separated subsets contain a single isolated extreme point of U. The function $|\nabla_z U(z)|$ is C^1 by construction, then $\nabla_z|\nabla_z U(z)|$ can be calculated and locally v can be selected proportional to $\nabla_z|\nabla_z U(z)|$ in order to maximize $|\nabla_z U(z)|$, which is equivalent of the extreme point avoidance. In the simple case presented above the calculation of $\nabla_z|\nabla_z U(z)|$ may be avoided.

4.6. Results of simulation

For $\upsilon = 0.1$, $\Upsilon = 0.5$, $\alpha = 2$, $N = 1$ and $(x_1, y_1) = (2,2)$ with $d_1 = 1$, the results of the system (7) simulation for different initial conditions with $v = 0$ are shown in Fig. 3a. The results of the system (7) simulation with (10) and (11) are shown in Fig 3b (the difference between the controls (10) and (11) is not visible in this scale). As we can conclude, for $v = 0$ the potential field method sticks in the local extreme for some initial conditions, while with the proposed modifications (10) or (11) the origin is attractive under provided restrictions.

5. Wheeled mobile robot regulation with obstacle avoidance

Consider a wheeled mobile robot, whose kinematic model is given by a unicycle:

$$\begin{aligned} \dot{q}_x &= \cos(\theta)u(1+\delta_1), \\ \dot{q}_y &= \sin(\theta)u(1+\delta_1), \\ \dot{q}_\theta &= \omega(1+\delta_2), \end{aligned} \tag{12}$$

where $(q_x, q_y) \in \mathcal{M}$ is the robot position and $\mathcal{M} \subset \mathbb{R}^2$ is a compact set containing the origin, $q_\theta \in (-\pi, \pi]$ is the robot orientation, $|u| \le u_{\max}$ and $|\omega| \le \omega_{\max}$ are linear and angular velocities of the robot respectively ($u_{\max}$ and $\omega_{\max}$ are given bounds), $\delta_k \in [\delta_{\min}, \delta_{\max}]$, $k = 1,2$ are exogenous bounded disturbances, which are introduced in order to represent the dynamical model uncertainties/dynamics (they are not taken into account in the usual unicycle model) [49] , $-1 < \delta_{\min} < \delta_{\max} < +\infty$.

The easiest way to apply the strategy to an unicycle-like WMR would be to linearize the system [25] considering the dynamics of a point Ψ on the x axis of the robot reference frame (Fig. 4) and apply the control (6) to it. In particular the point $\Psi = (\Psi_x, \Psi_y)^T = (q_x + \psi\cos q_\theta\,,\ q_y + \psi\sin q_\theta)^T$, with ψ the distance between the robot center and Ψ, has the following dynamics:

$$\begin{bmatrix} \dot{\Psi}_x \\ \dot{\Psi}_y \end{bmatrix} = R \begin{bmatrix} u(1+\delta_1) \\ \omega(1+\delta_2) \end{bmatrix},\ R = \begin{bmatrix} \cos q_\theta & -\psi sin q_\theta \\ \sin q_\theta & \psi\cos q_\theta \end{bmatrix}. \tag{13}$$

The expression (13) can be rewritten as

$$\begin{bmatrix} \dot{\Psi}_x \\ \dot{\Psi}_y \end{bmatrix} = R \begin{bmatrix} u \\ \omega \end{bmatrix} + R \begin{bmatrix} u\delta_1 \\ \omega\delta_2 \end{bmatrix}. \tag{14}$$

Let us consider $R\begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$ of (6) and (11), the following theorem will apply:

Theorem 5. *Let Assumption 1 to be satisfied. The system (14) with control (6) and (11), where $\varepsilon > 2$, $\epsilon > 0$ sufficiently small, has the origin globally attractive, provided that* $\|\delta\| < \max\left\{\psi, \psi^{-1}\right\}$.

Proof. Let us consider that $\left|R\begin{bmatrix} u\delta_1 \\ \omega\delta_2 \end{bmatrix}\right| \le |d|\,|R|\left|\begin{bmatrix} u \\ \omega \end{bmatrix}\right|$. Being $\begin{bmatrix} u_x \\ u_y \end{bmatrix} = -\nabla_\Psi U(\Psi) + v = R\begin{bmatrix} u \\ \omega \end{bmatrix}$, and considering $V = U(\Psi)$ as a Lyapunov function, with $U(\Psi)$ defined as (5), then:

$$\begin{aligned} \dot{V} &= \nabla_\Psi U(\Psi)\left(-\nabla_\Psi U(\Psi) + v + R\begin{bmatrix} u\delta_1 \\ \omega\delta_2 \end{bmatrix}\right) \\ &\le -|\nabla_\Psi U(\Psi)|^2 + |\nabla_\Psi U(\Psi)|\,v + |R|\left|R^{-1}\right||d|\,|\nabla_\Psi U(\Psi)|^2 \\ &\le \left(|R|\left|R^{-1}\right||d| - 1\right)|\nabla_\Psi U(\Psi)|^2 + |\nabla_\Psi U(\Psi)|\,v \end{aligned}$$

it follows that if $|R|\left|R^{-1}\right||\delta| < 1$ the stability is proven using the results of Lemma 2 and Theorem 4.

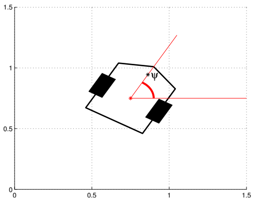


Figure 4: Position of the Ψ point.

Remark 1. The value $|R|\left|R^{-1}\right||\delta|$ can be rewritten as $\max\left(\psi,\psi^{-1}\right)|\delta|$, it means that selecting ψ carefully the condition is always verified.

Such a technique is easy and effective as can be seen in Fig. 5 (blue path), but it doesn't allow the direct control on the WMR orientation and on the positivity of the linear velocity, resulting for instance in backward maneuvers (this happens when the initial conditions are not ideal, like the WMR non facing the target point). The authors want to avoid this kind of movements for practical reasons.

For this purpose, to control both linear velocity and orientation of the WMR the trajectory generated by (7), (11) (or with (10)) can be used as a reference for (12), defining $\theta_d = \operatorname{arctan2}\left(\nabla_y U(z), \nabla_x U(z)\right)$ and $\gamma = \theta_d - q$:

$$u = \frac{u_{\max}}{1+\varepsilon}\sqrt{u_x^2+u_y^2}, \qquad (15)$$
$$\omega = \left(\omega_{\max}\sqrt{|\gamma|}+\bar{k}\right)\operatorname{sign}(\gamma).$$

Theorem 6. *Let Assumption 1 hold. The control (15) stabilizes the $\gamma(t)$ variable in finite-time orienting the robot as the gradient $\nabla_z U(z)$ of the field , it follows that the system (12) with control (15) has the origin globally attractive.*

Proof. Let us consider the variable $\gamma(t)$, and consider the Lyapunov function $V = \frac{1}{2}\gamma^2$, then:

$$\dot{V} = \gamma\left[\dot{\gamma} - \omega\left(1+\delta_2\right)\right].$$

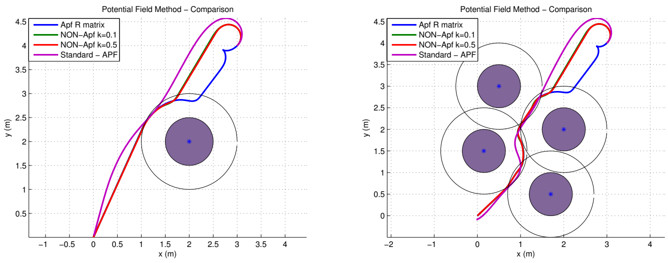
As specified in section 4, the $U(z)$ field is a C^1 function therefore the derivative of $\gamma(t)$ exists, is continuous and bounded because of the construction of the field $U(z)$. Such a derivative is bounded, then it is possible to find a

$$\bar{k} \geq \left(\frac{d}{dt}\left(\operatorname{arctan2}\left(\nabla_y U(z), \nabla_x U(z)\right)\right)\right) / \left(1-\delta_{min}\right)$$

to have $\dot{V} \leq 0$. Since, of consequence, $\exists\ T \geq t_0$ time in which the robot orientation is aligned with one of the gradient lines, under Assumption 1 and Theorems 2 and 4, the controller u in (15) asymptotically stabilizes the WMR.

5.1. Simulations

The results of simulation for the system (14) with control (6) and (11) and for the system (12), (15) are shown in Fig. 5. The bounds for the inputs are $u_{\max} = 1$ and $\omega_{\max} = 3$. Two cases are presented: single obstacle (Fig.5a) and multiple obstacles (Fig. 5b). In both parts of Fig. 5 the obstacle is the zone filled in violet, while the distance of influence is the black circle around the obstacle itself. The proposed methods are called *Apf-R matrix* and *NON-Apf*, the latter to emphasize the *non* asymptotic (finite time) behavior of the controller acting on the orientation of the WMR. Moreover the proposed modifications have been compared with the standard APF [14].



(a) Single obstacle (b) Several obstacles

Figure 5: The result of simulations for the three different modification of the PF method.

In both figures of Fig. 5 the comparison with the standard APF has been made for 2 values of $\bar{k}$, when using the *NON-Apf* control, to show how it affects the control inputs and the overall performances.

The unwanted behavior of the controller *Apf-R* matrix method discussed in the previous section is visible both in Fig. 5 and Fig. 7; the path followed by the WMR (Fig. 5) using this method clearly shows a backward maneuver, as it is confirmed from Fig. 7, where it is shown the negative linear velocity input.

Fig. 6 shows the orientation of the robot q_θ with respect to the direction of the field θ_d, the desired angle, as the $\bar{k}$ gain changes (not controlling the orientation of the WMR the *Apf-R matrix* method has been omitted from the plot). The second column of Fig. 6 shows how the controlled variable γ evolves. As it can be gathered from the plots, as the value of $\bar{k}$ increases the WMR reacts faster to the change of direction of the field due to the obstacles presence, decreasing also the instantaneous value of the γ error variable. Nevertheless, these improvements come with a drawback, increasing $\bar{k}$ (see $\bar{k} = 0.5$, Fig. 6 and Fig.7) could cause a bit of chattering around the stabilization point due to the increased control effort as it can be noticed also in Fig. 7.

5.2. More complex scenarios

Several simulations were run also for more complex scenarios, in which the features of the equipped sensor for the implementation are taken into account. In the case the real WMR has a LIDAR laser ranger finder. In Fig. 8b is shown the path followed by the WMR using the proposed modification of the potential field, while in Fig. 8a the strategy to decide which is the "point" to use as reference for the obstacle. Basically, the chosen point ζ, green star in Fig. 8a, is the averaging on the x and y coordinates of the LIDAR sensed points in a predefined range; the radius is the distance among ζ itself and the farthest sensed point of the scan, which leads to the definition of the in-

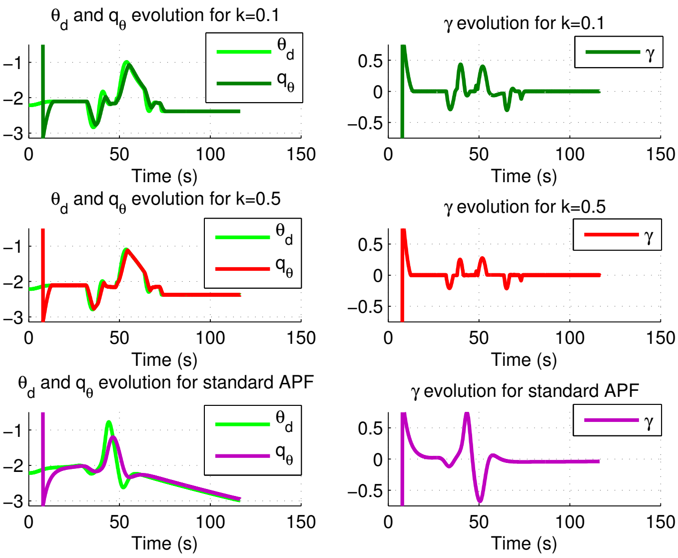


Figure 6: Evolution of the WMR orientation q_θ and desired angle θ_d and respective error variable dynamic γ

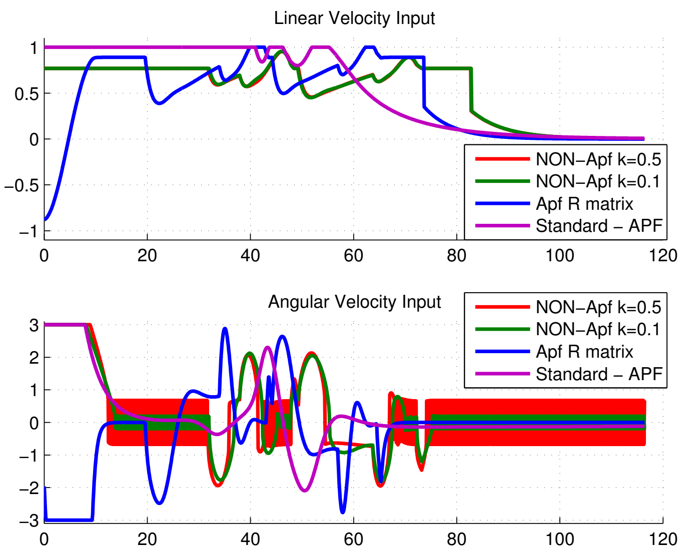
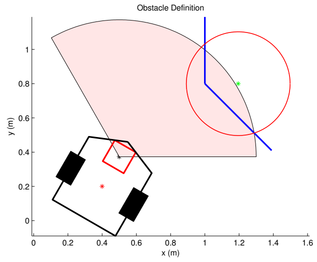
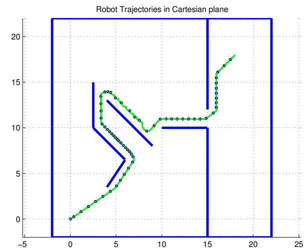


Figure 7: Input Signals for the different methods

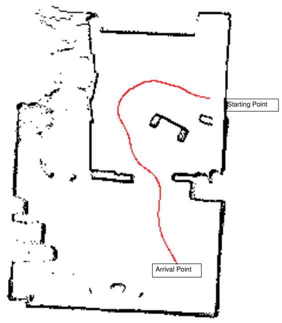


(a) WMR equipped with a LIDAR laser range finder: obstacle definition

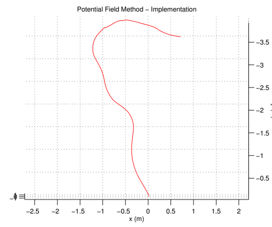


(b) The path followed by the WMR in a complex scenario

Figure 8: Results on complex environment



(a) Trajectories in the map



(b) Trajectories in Cartesian plane

Figure 9: The trajectories followed by the WMR in a real environment

fluence distance d that is the radius augmented of the diameter of the robot.

5.3. Implementation

The presented strategy has been implemented on a Turtlebot2 (`http://www.turtlebot.com/`) mobile robot. The WMR was equipped with a Hokuyo® (`http://www.hokuyo-aut.jp`) UTM-30LX LIDAR device. The necessary libraries to communicate with the WMR were found on Robotic Operating System (ROS) (www.ros.org). The same strategy used in Section 5.2 to simulate the LIDAR based obstacle detection algorithm has been implemented to get obstacles positions $\zeta_i = (x_i, y_i)$ in real time. The WMR avoided obstacles without any previous knowledge of the environment, nevertheless some oscillations have been noticed while moving in narrow corridors. The trajectories followed in an office-kind environment are shown in Fig.9a, the WMR objective is to reach the origin of the global frame in the lower-right corner, a plot of the trajectories in the Cartesian plane is given in Fig. 9b while Fig. 10 shows the evolution of the q_x and q_y through the origin. As it can be seen the robot avoids the obstacles and passes trought a narrow passage ($80cm$) to finally arrive to its destination (it is useful to remark that the robot has no knowledge of the obstacles a priori).

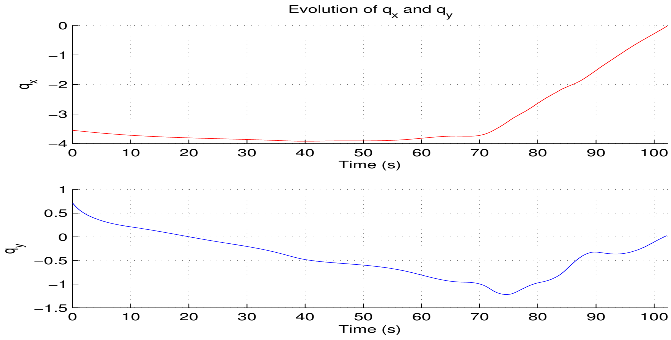


Figure 10: Evolution of the $q_x$and $q_y$variables.

6. Conclusions

Local minima represent a problem when applying the potential field method. This work presented a solution to avoid them, using a control theory result [19], which allowed the authors to prove the ISS property for a system of two integrators fed with the gradient of an *ad-hoc* designed field. It has been formally shown how the introduction of a small perturbation v as input does not introduce new equilibriums, making the origin the only attractive point for the system. Two different strategies have been proposed and proven to apply the method to the obstacle avoidance problem for an unicycle-like WMR. The first strategy, linearize the output to directly apply the results synthesized for the particle case, without having the capability to control the orientation of the robot. A second strategy is, thus, presented which uses the particle case results as a base to design a control. The control steers the robot in the direction of the field lines in finite time. Both formulations are presented in simulations for a unicycle-likle WMR, it is shown how the task is achieved avoiding standing alone and multiple obstacles and in complex environments. Real experiments with a Turtlebot II platform in an office environment are presented too, showing some issues in presence of narrow passages and obstacles excessively close to the target, which did not prevent to reach the goal.

The authors intend to improve the method to cancel any oscillations, to augment the dimension to the 3D case and to extend it to be used in the case of multi-agent systems.

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